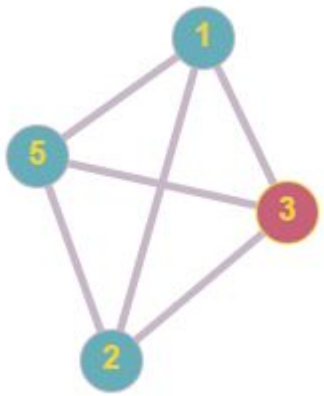
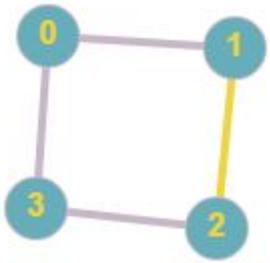
Meyniel’s conjecture

Cops and robbers





Cop number

- NP hard
- Parameterized version W[2] hard

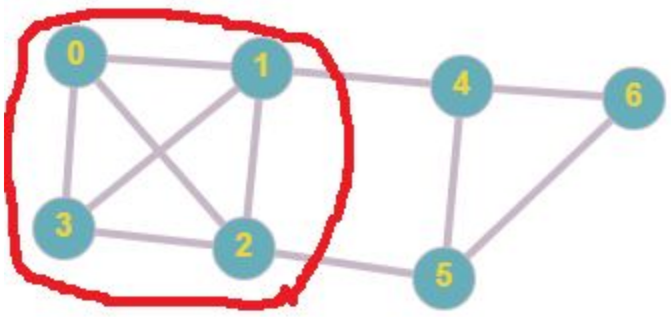
Meyniel's conjecture

$$O(\sqrt{n})$$

Soft Meyniel's conjecture

$$O(n^{1-c})$$

Guarding subgraphs

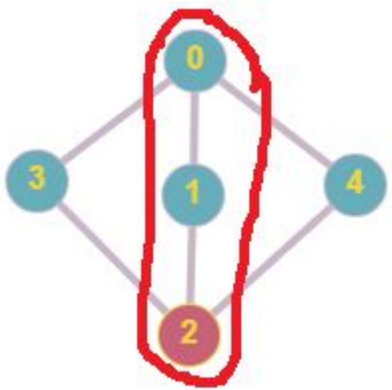


Induced subgraph H is called k-guardable if k cops are are enough to guard it completely.

Closed neighbourhood of a vertex is 1-guardable.

Isometric paths

- A path P of G is called isometric, iff for every pair of vertices on P, shortest distance between then in G can be realized with only edges from P
- Isometric paths are 1-guardable!



Moore bound

n - order, Δ - max degree, D - diameter

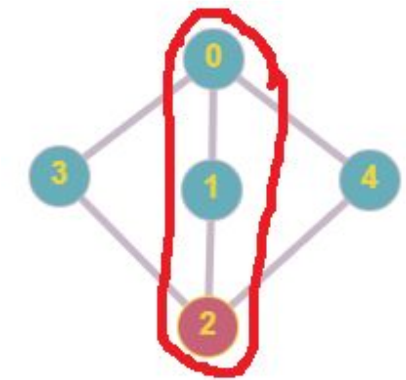
$$n \leq 1 + \Delta(\frac{(\Delta - 1)^D - 1}{\Delta - 2})$$

As a consequence, D,Δ = $\Omega(\frac{log\ n}{log\ log\ n})$

First bound

$$O\left(n^{\frac{\log \log n}{\log n}}\right)$$

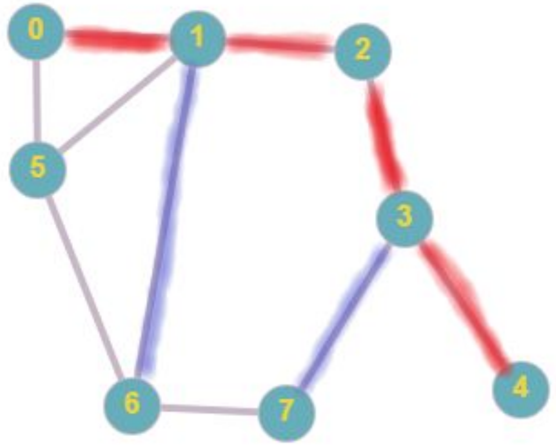
Greedy - find biggest isometric path or closed neighbourhood, place a cop there and run recursively on the rest of graph



Minimum distance caterpillar

Induced subgraph H st:

- H is a tree
- There is a path P in H st. distance to P from every vertex of H is <= 1



Minimum distance caterpillar

- Caterpillars are 5-guardable
- In every graph there is caterpillar of order at least log(n)

Second bound

$$O(\frac{n}{log\ n})$$

Same as previously but using caterpillars.

Best known bound

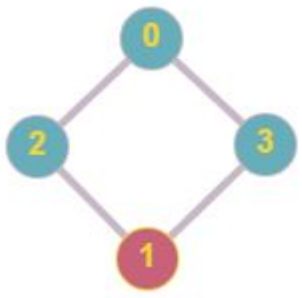
$$O\left(\frac{n}{2^{(1-o(1))\sqrt{\log_2 n}}}\right)$$

Random graphs

We measure expected cop number for random graph G(n,p).

For p sufficiently large Meyniel’s bound holds for random graphs.

Graphs with diameter 2



$$\leq 2\sqrt{n} - 1$$

Lower bound is tight

Family of graphs was found, for which cop number is

$$\geq \sqrt{\frac{n}{8}}$$

Open problem if it’s the best lower bound.