# Reconfiguring Independent Sets On Interval Graphs

Marcin Briański, Stefan Felsner, Jędrzej Hodor and Piotr Micek

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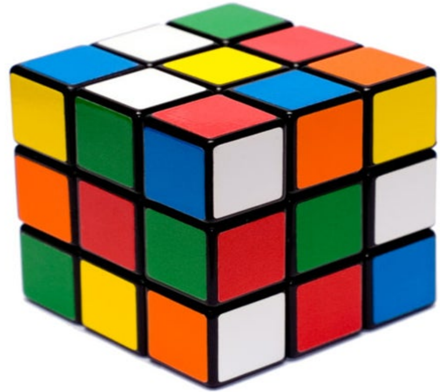

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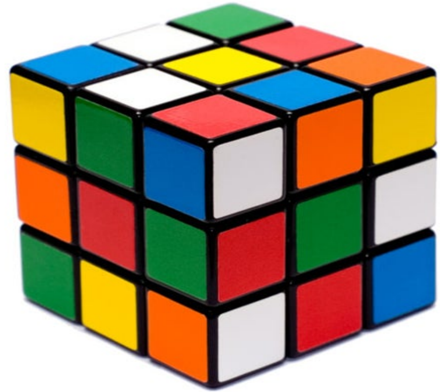

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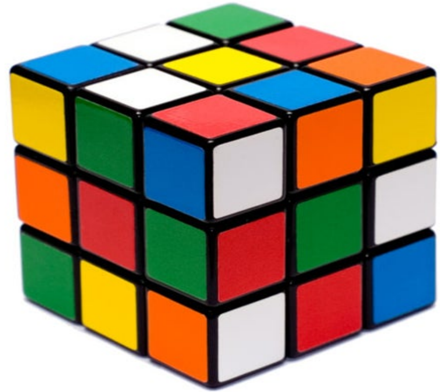

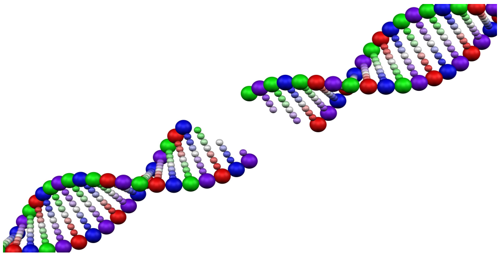

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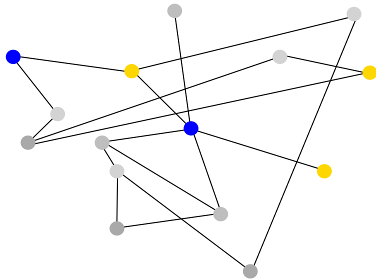

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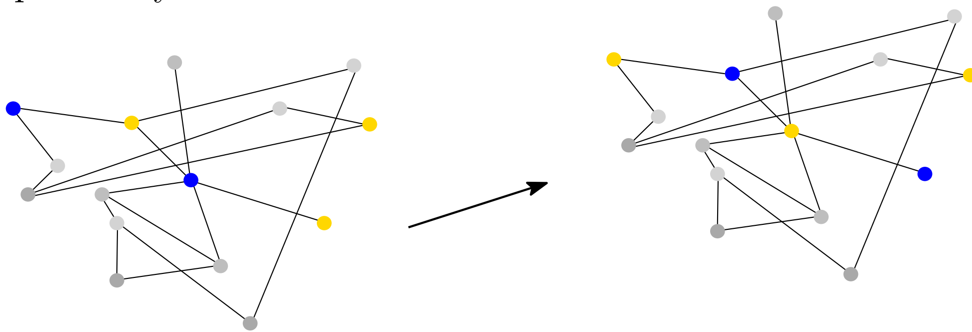

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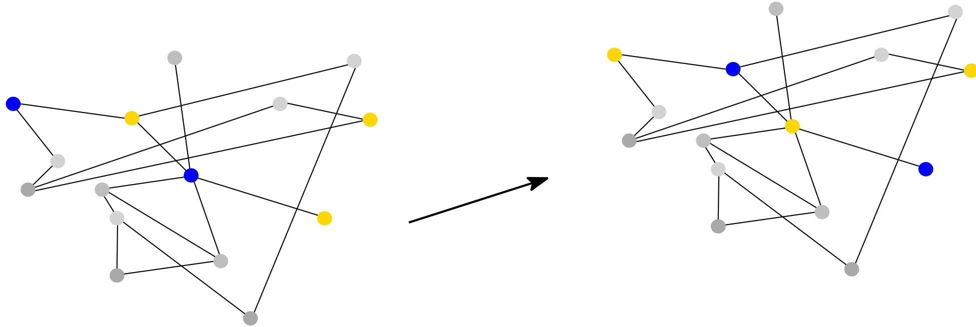

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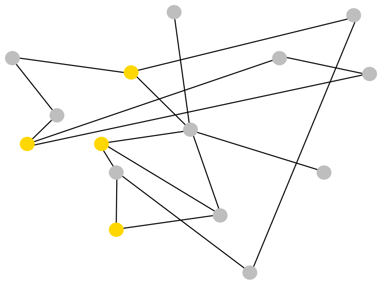

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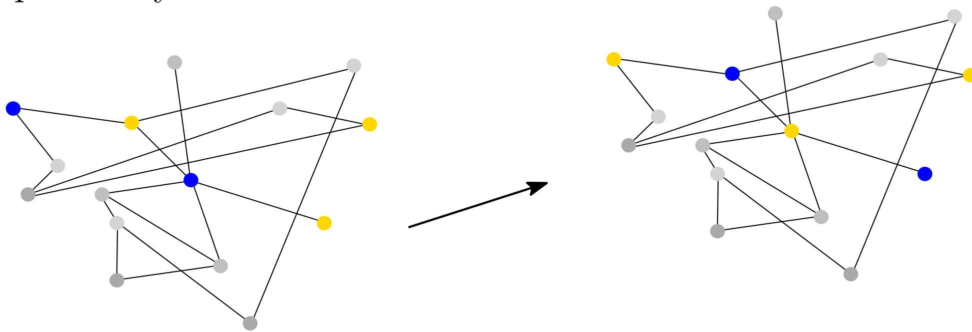

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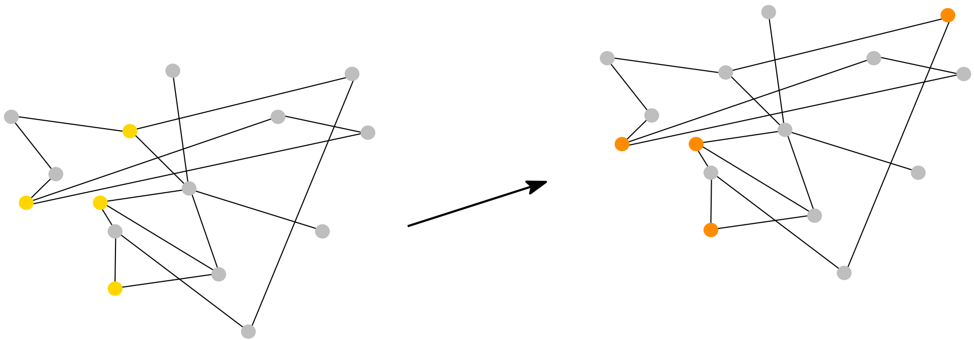

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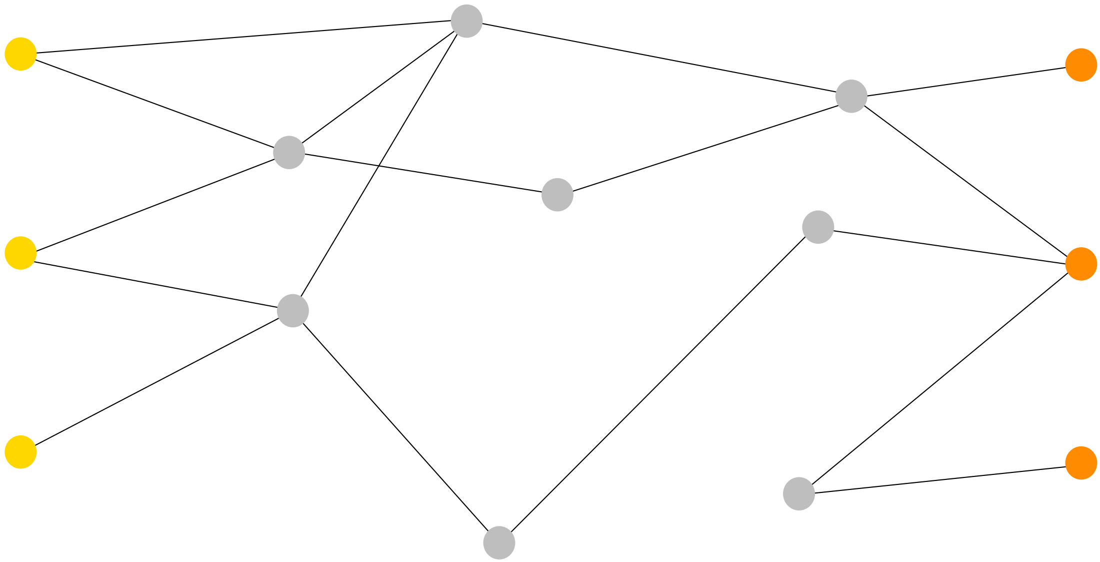

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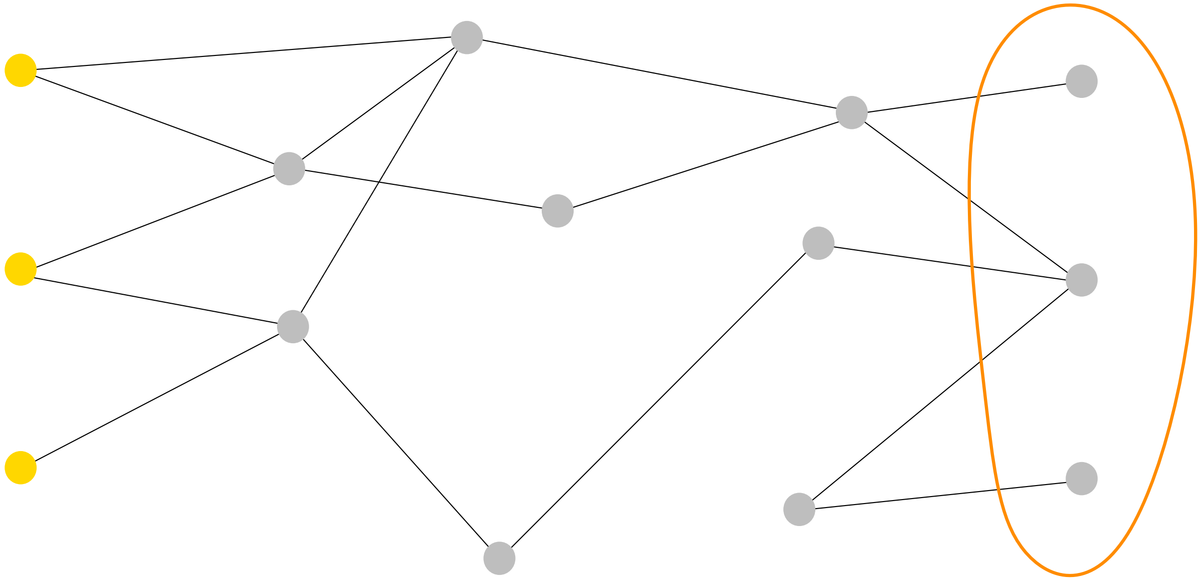

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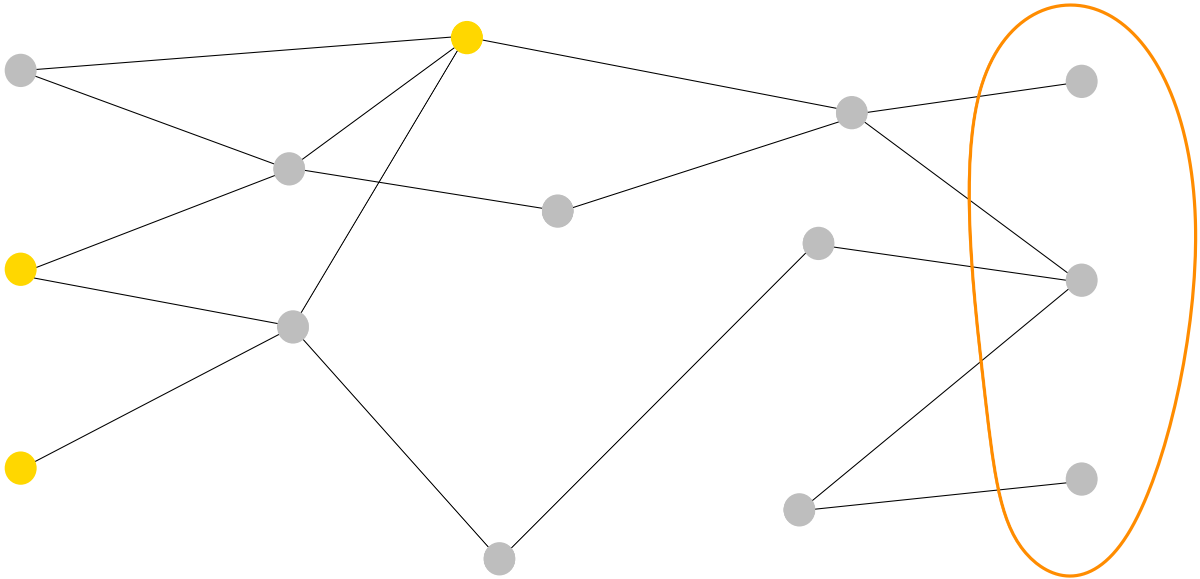

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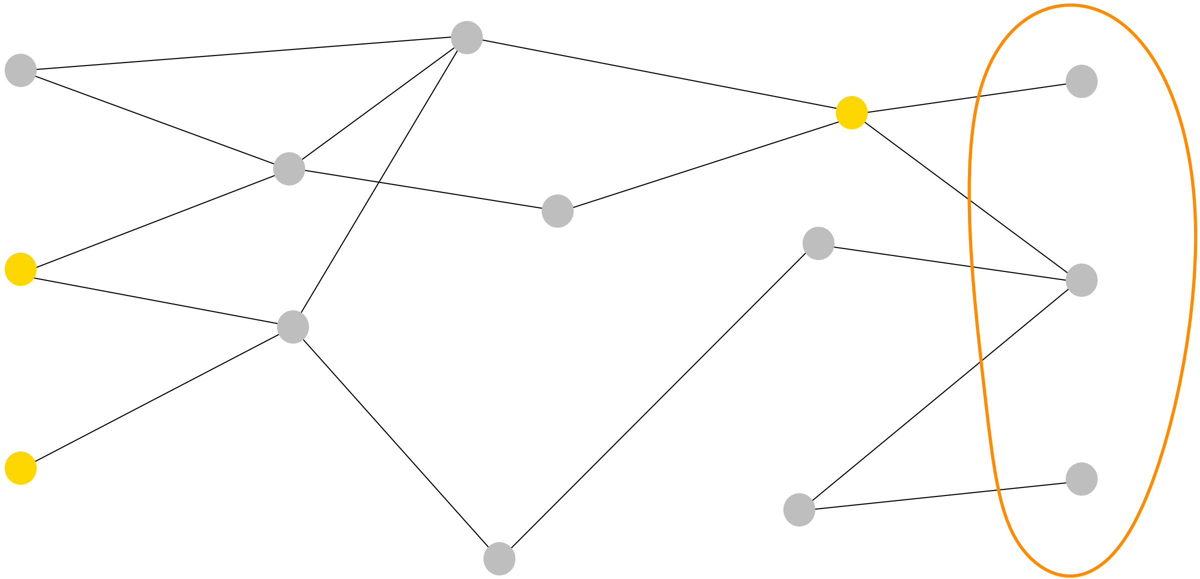

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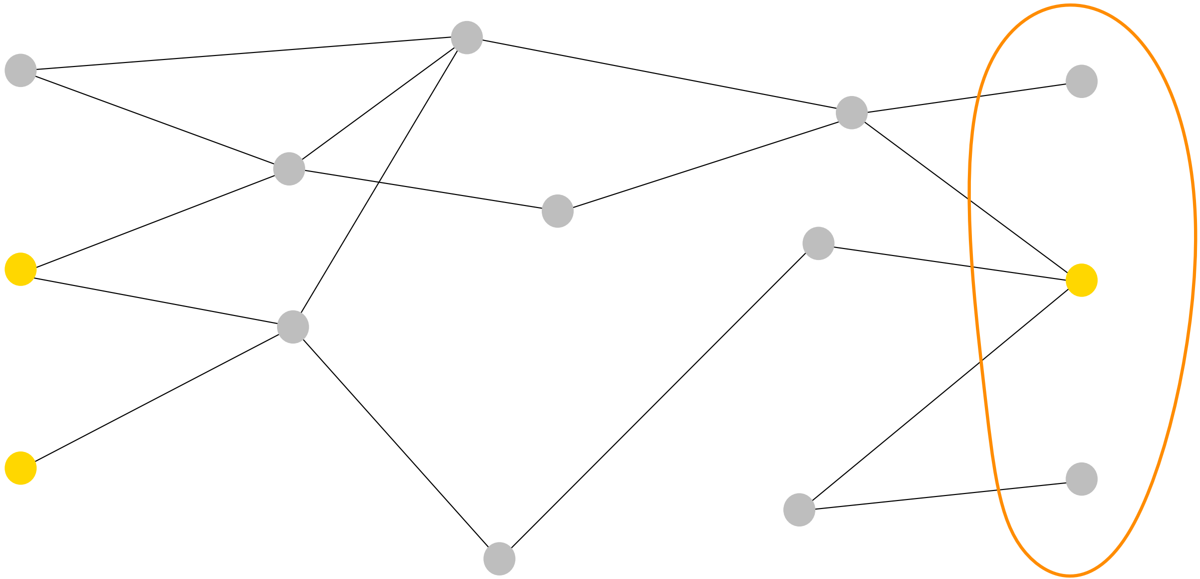

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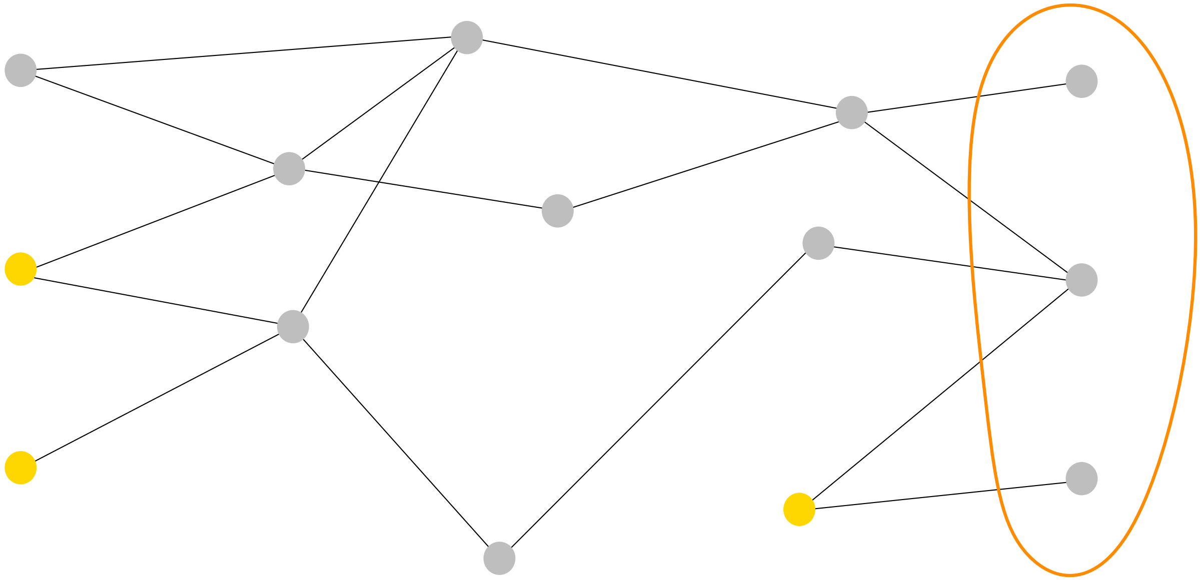

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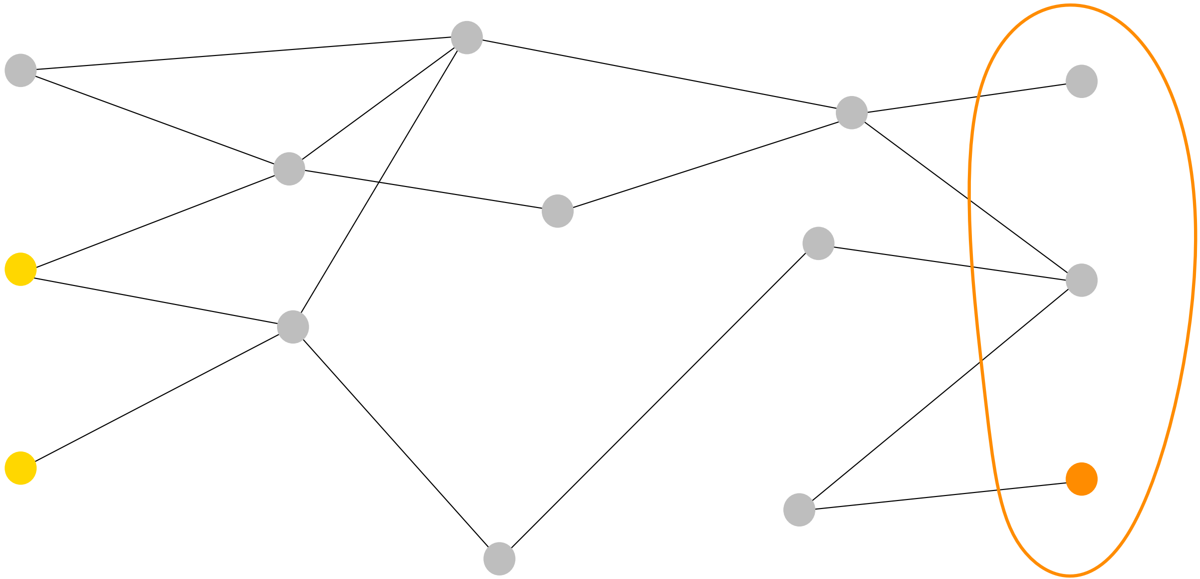

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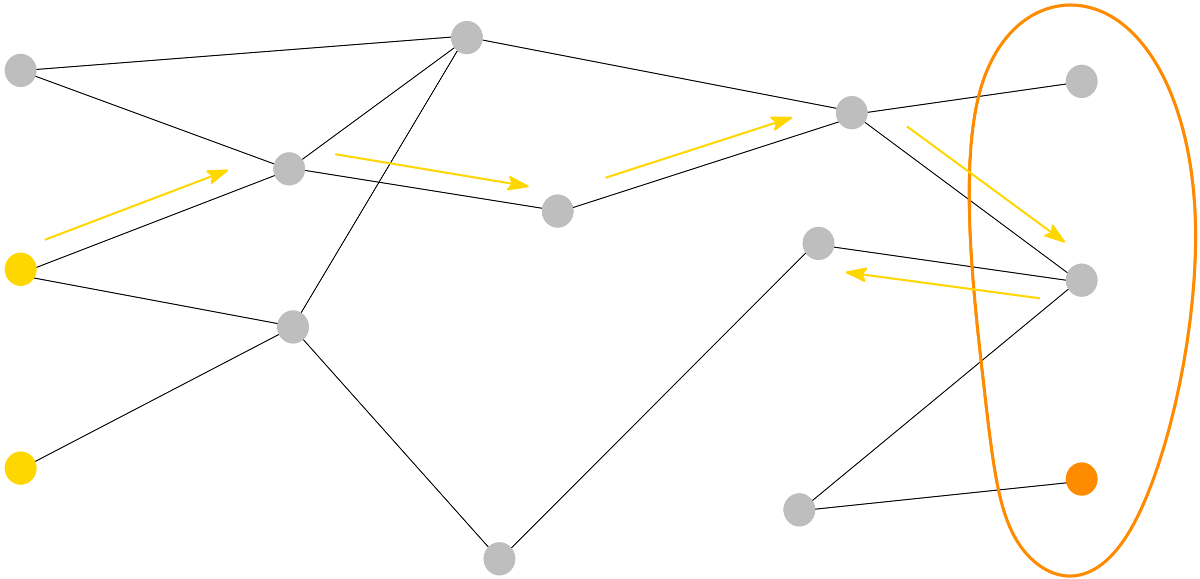

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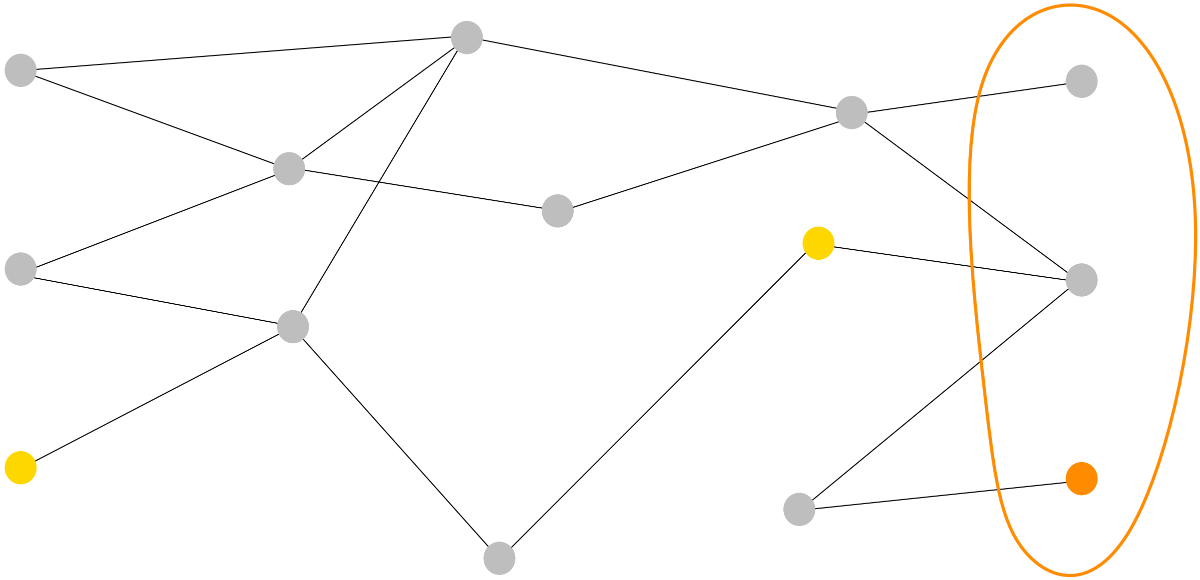

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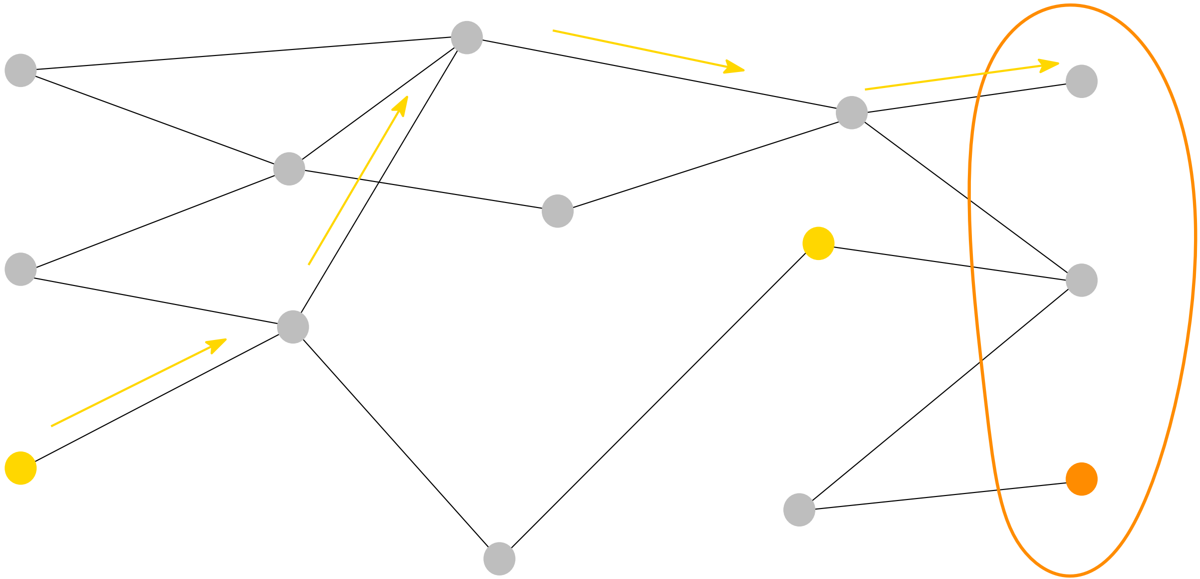

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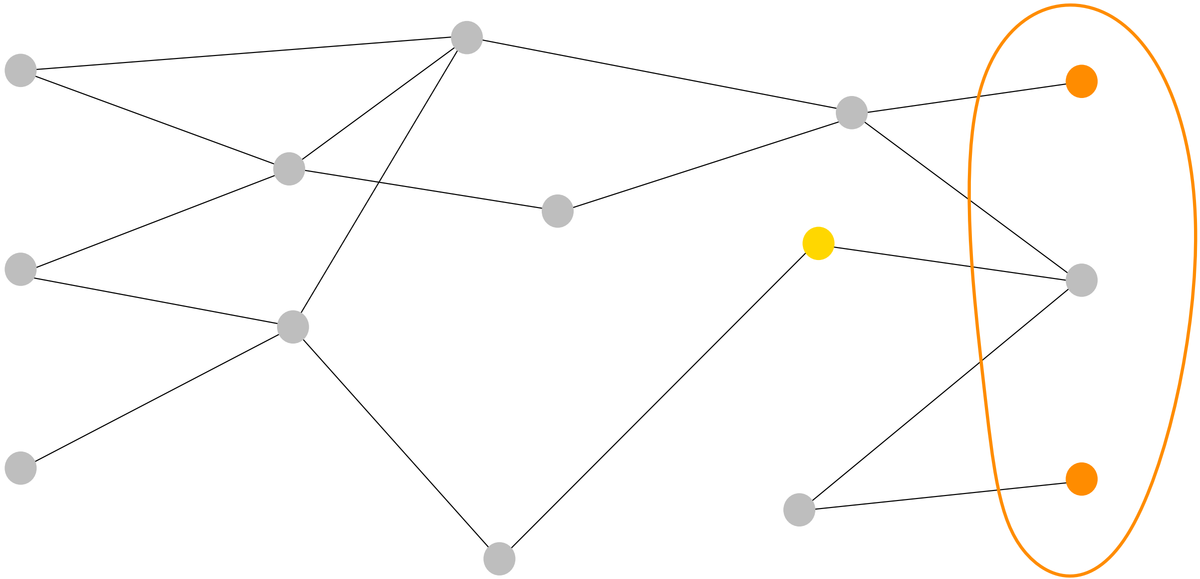

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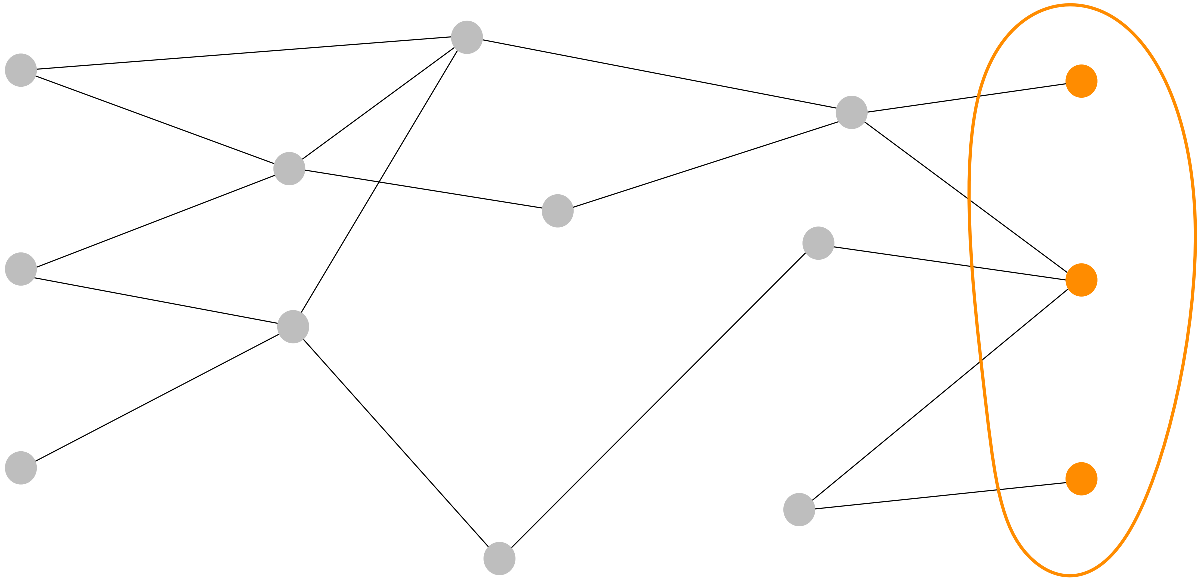

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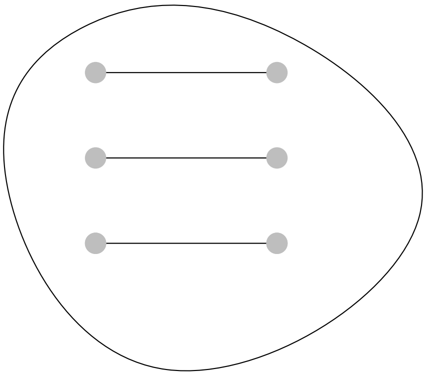

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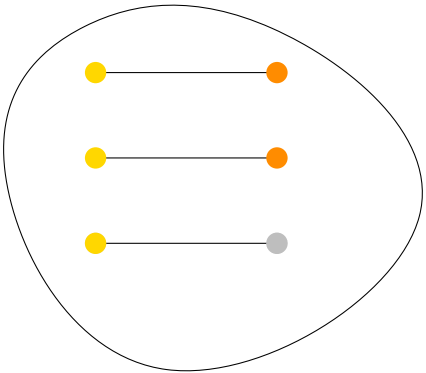

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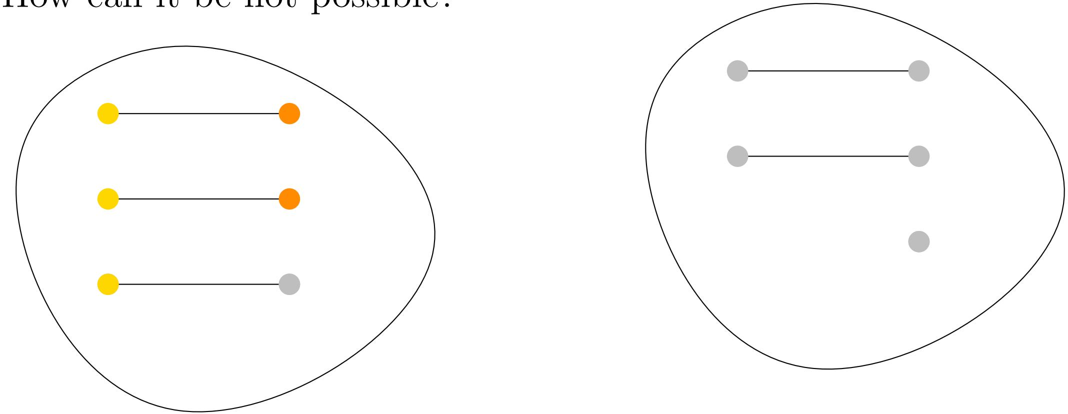

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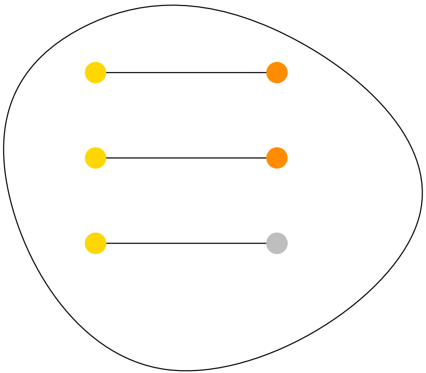

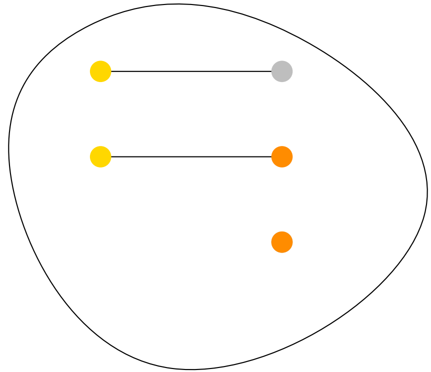

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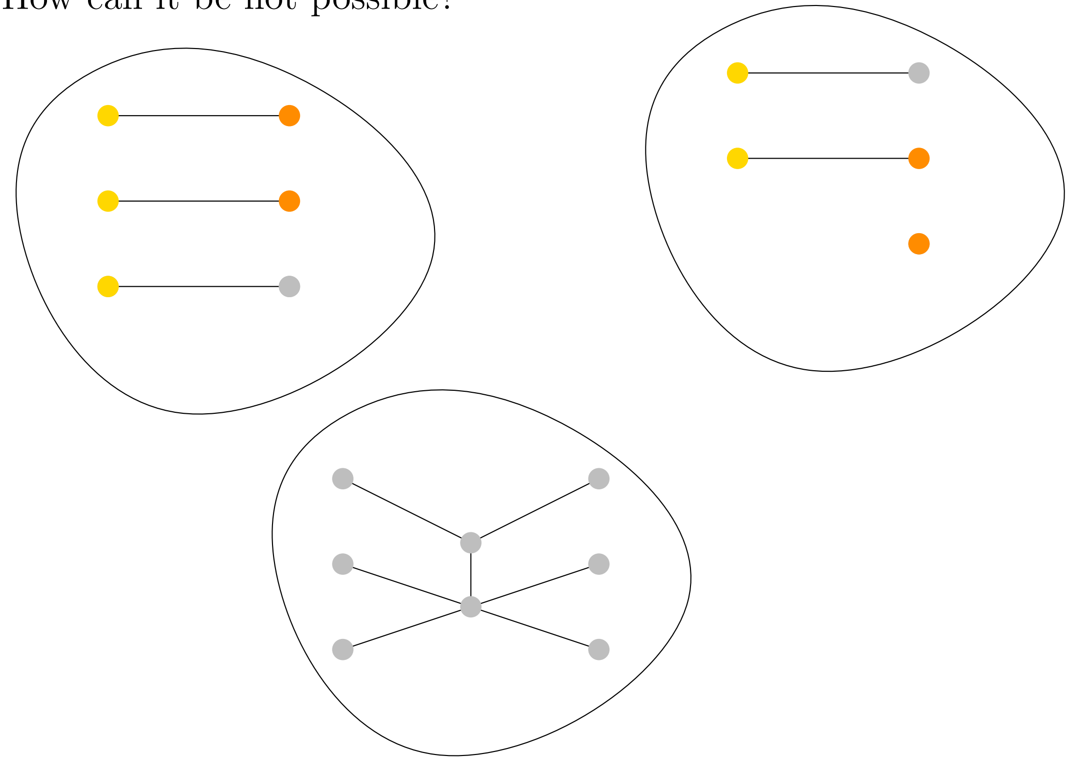

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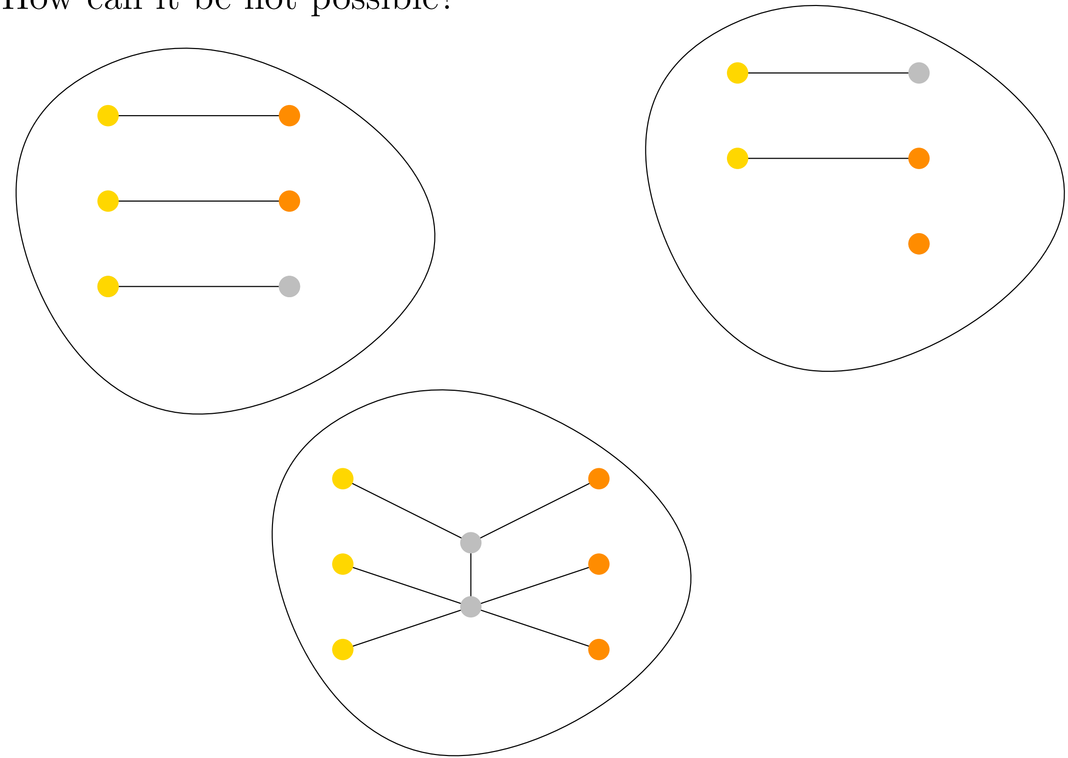

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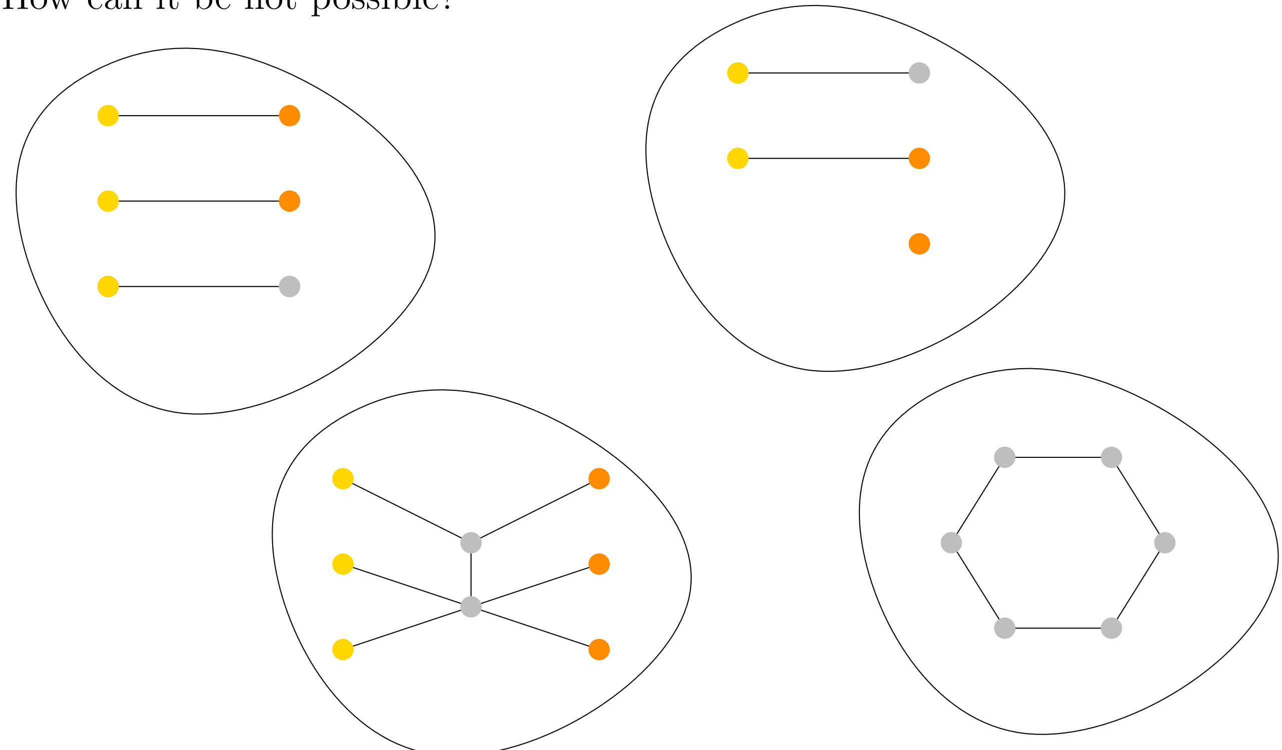

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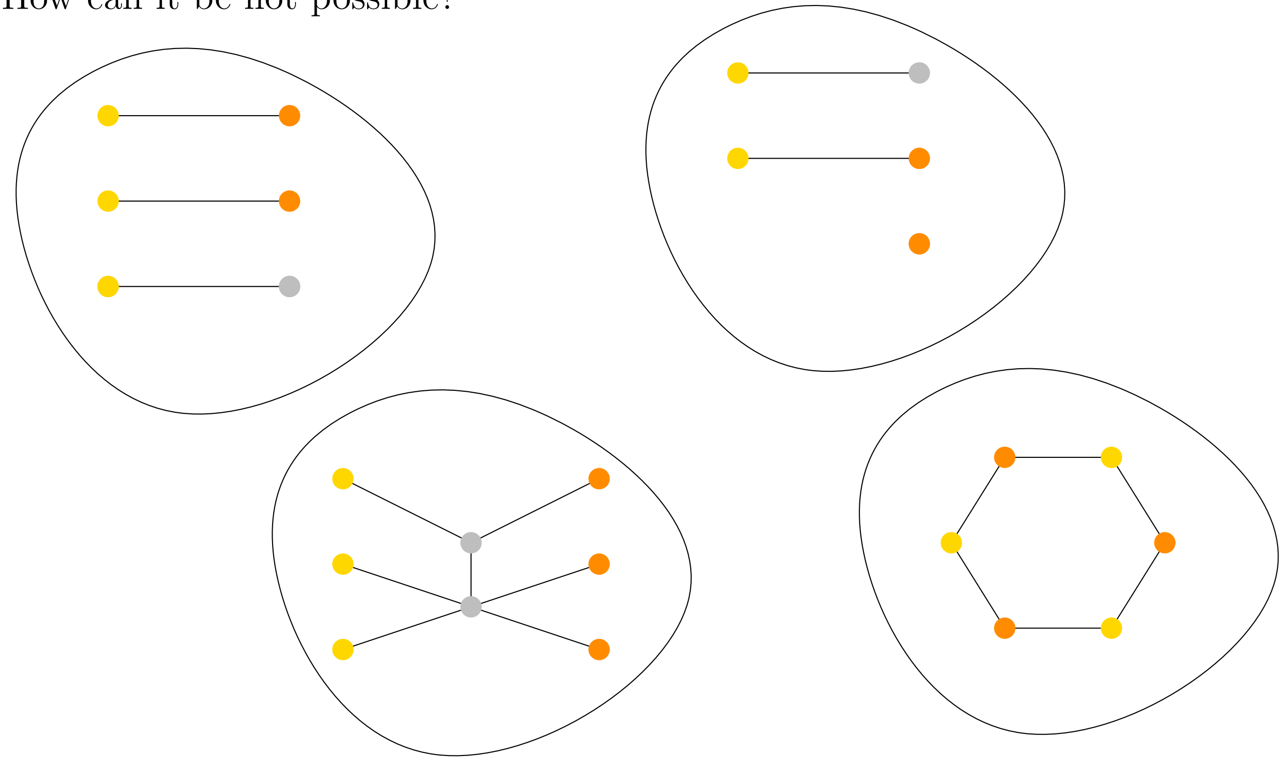

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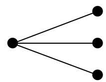

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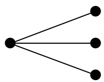

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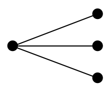

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- dynamic algorithm
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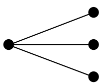

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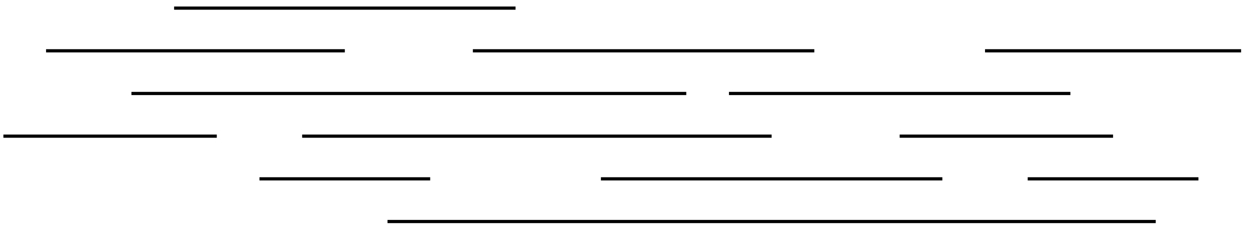

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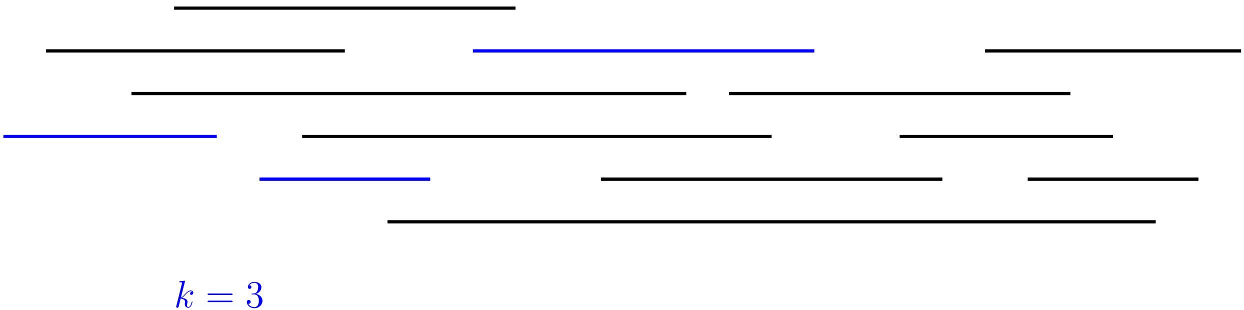

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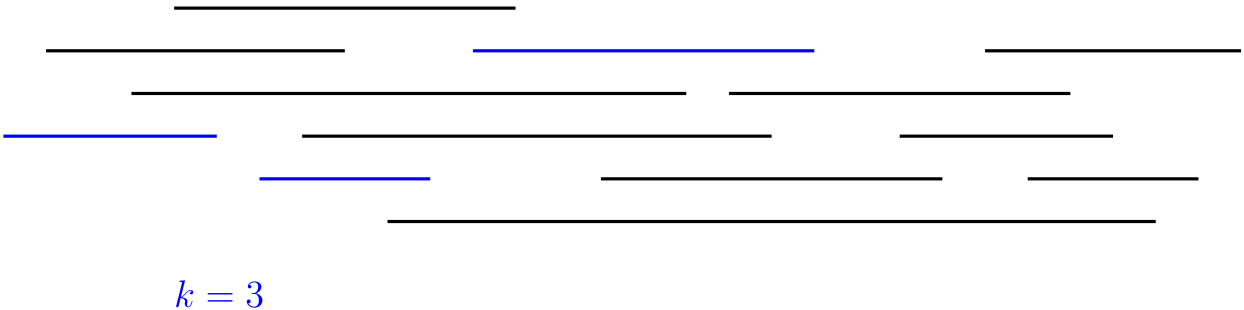


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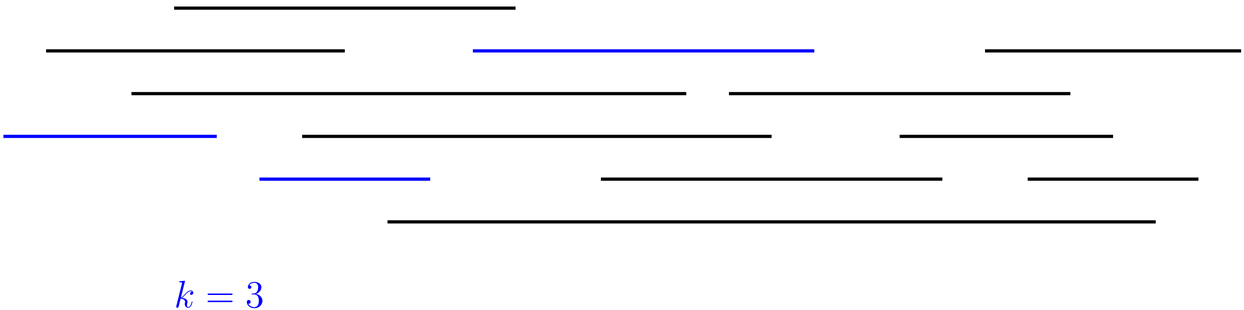


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$\text{EX}(\mathcal{C}) = \{\text{ex}_1(\mathcal{C}), \text{ex}_2(\mathcal{C}), \ldots, \text{ex}_k(\mathcal{C})\}$ ← "try to go left as far as you can"

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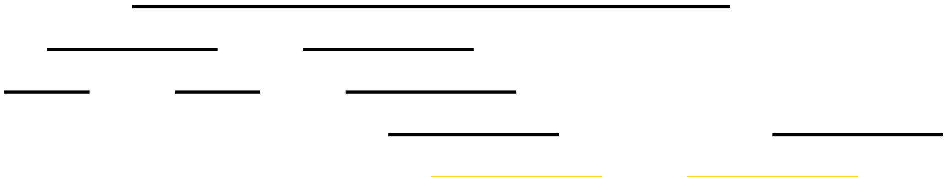

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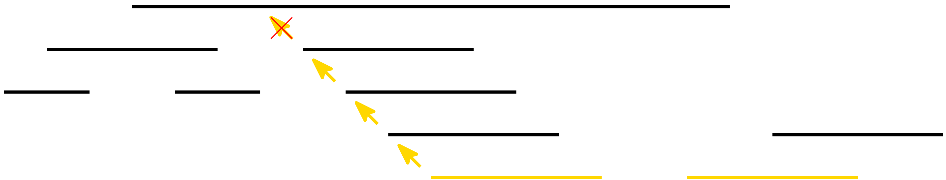

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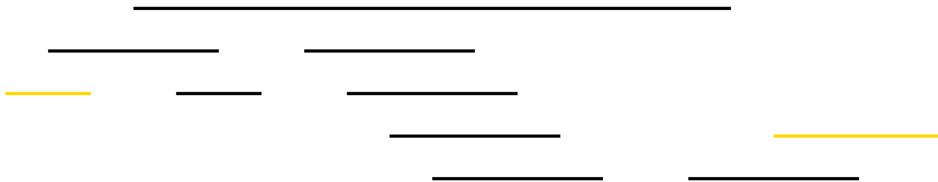

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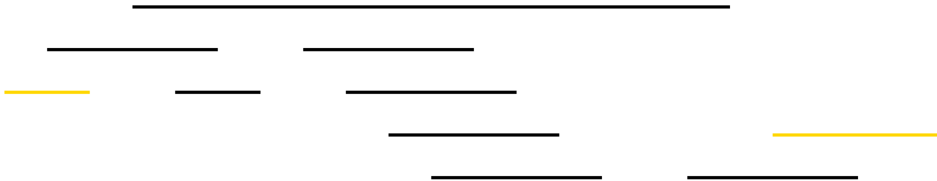

$\textsc{PushLeft}(l)$, $\textsc{PushRight}(r)$, $\textsc{PushLeft}(l)$,...

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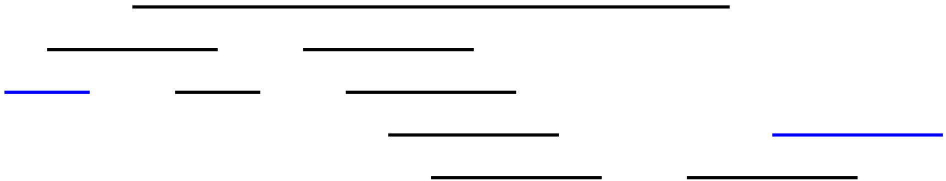

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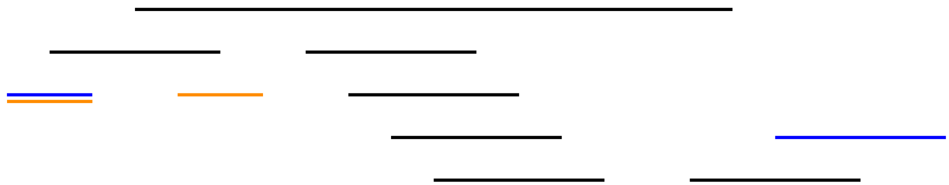

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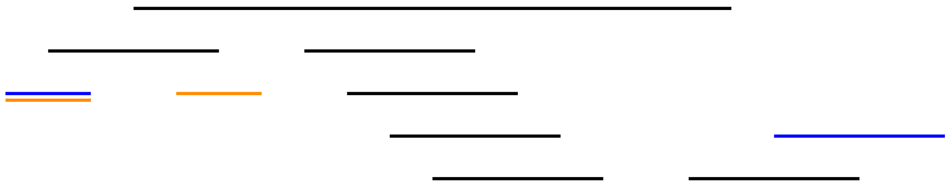

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The length of generated path is $\mathcal{O}(n)$

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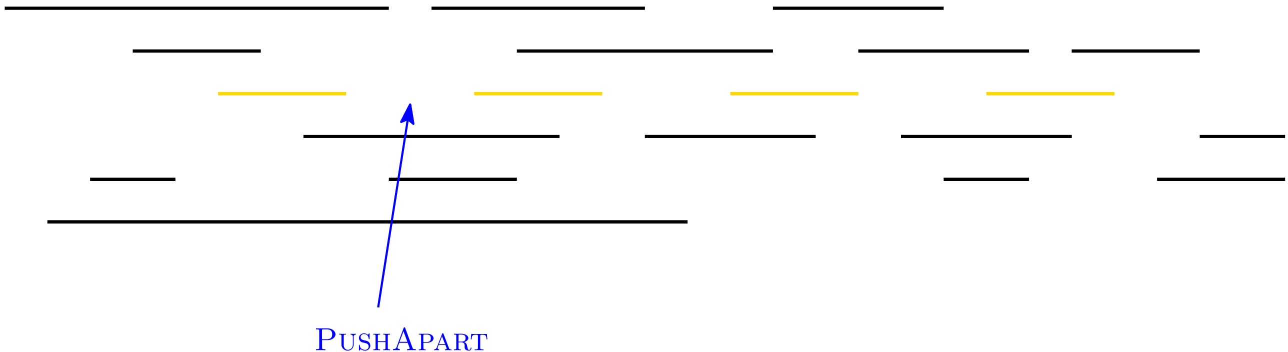

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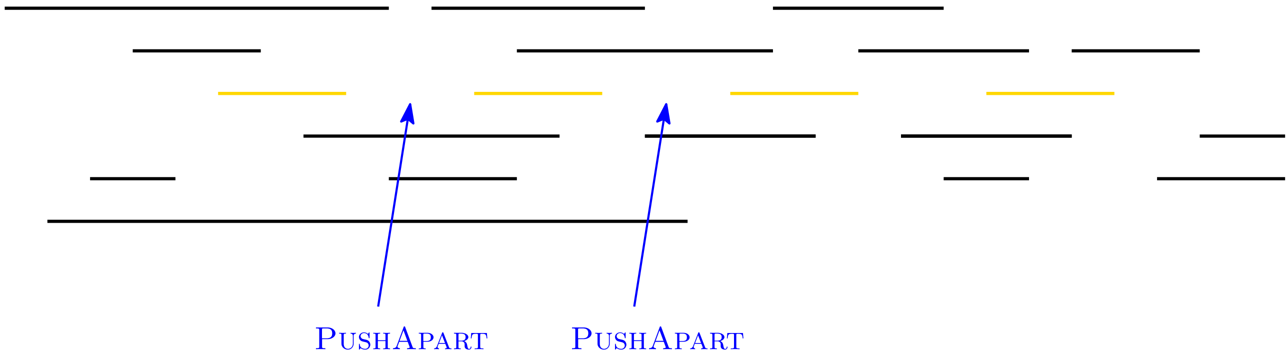

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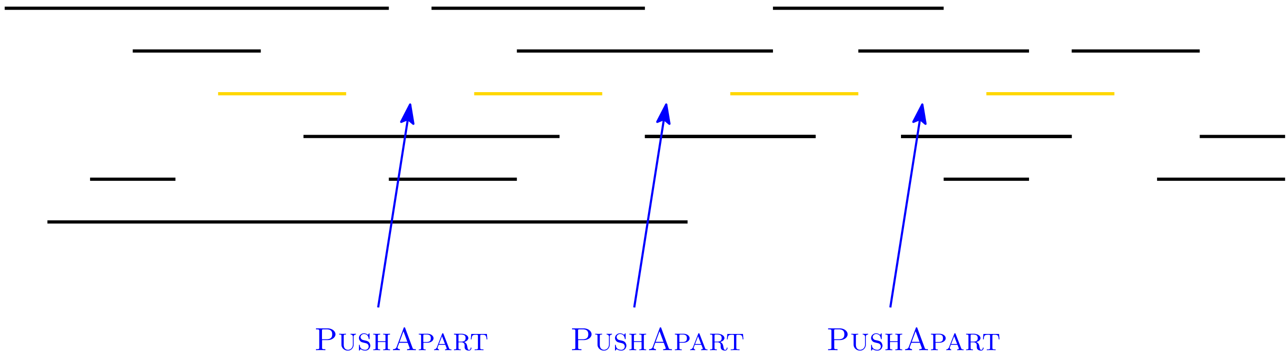

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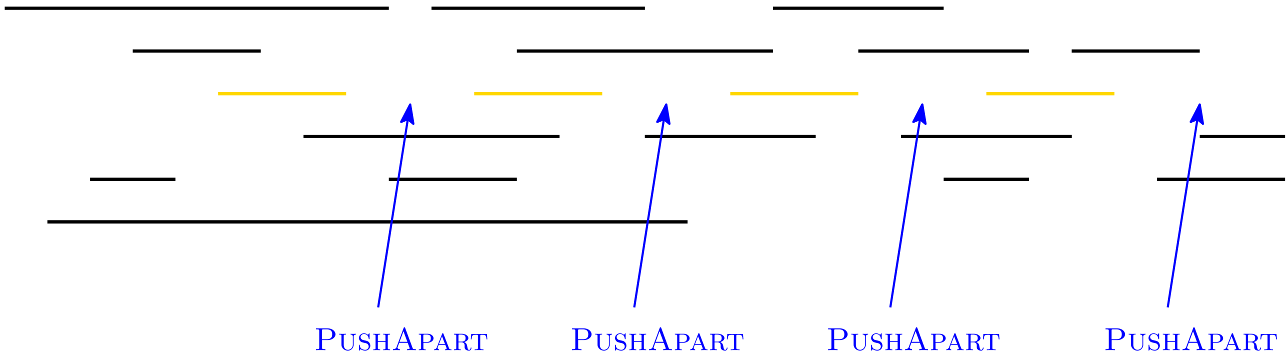

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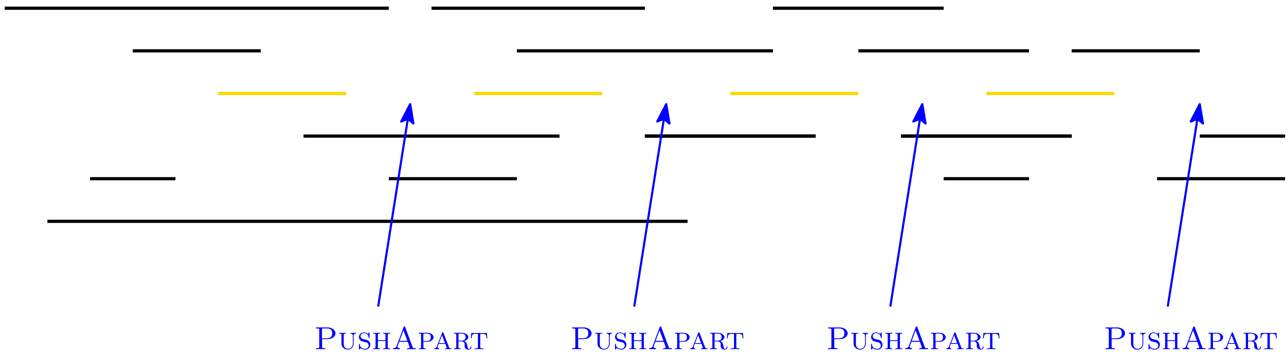


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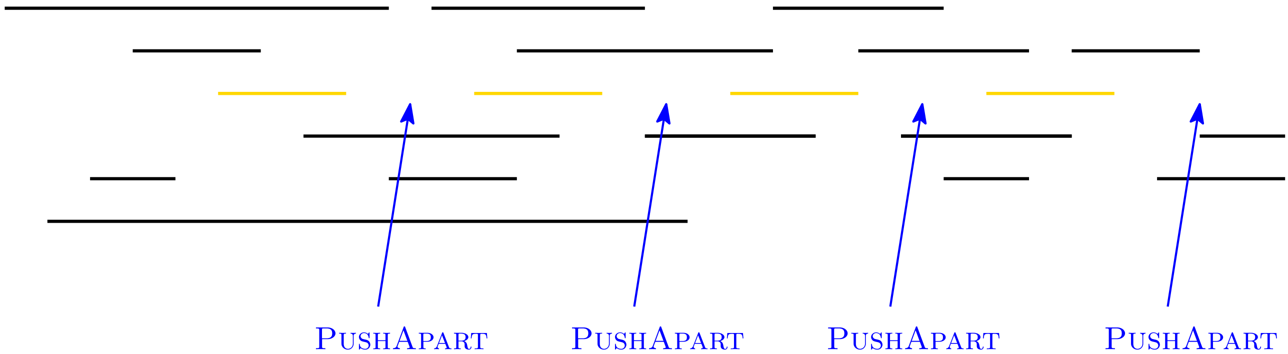


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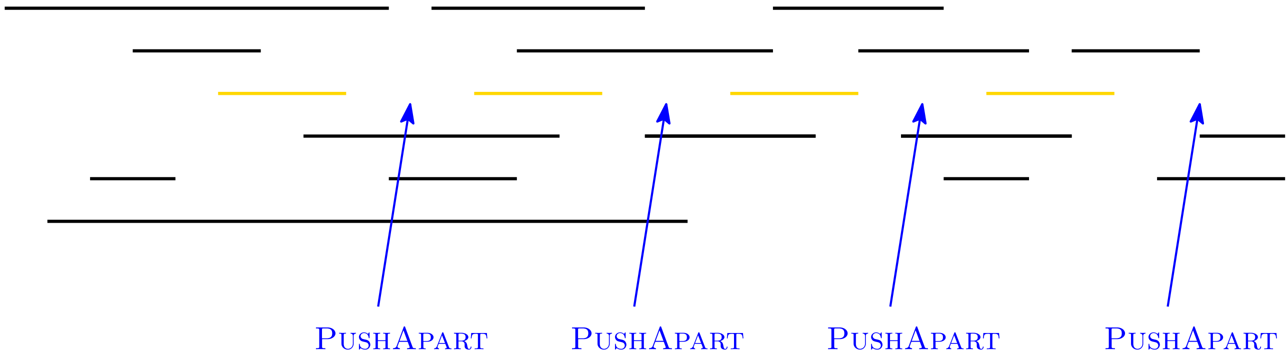


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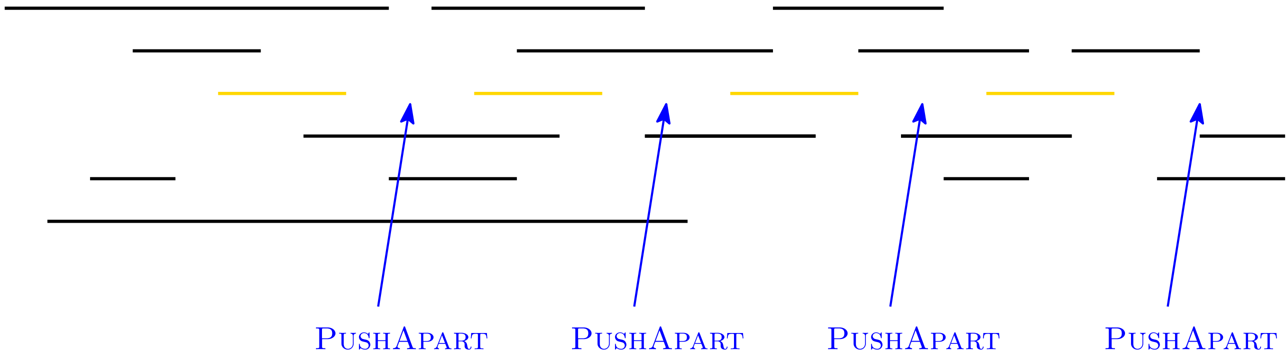


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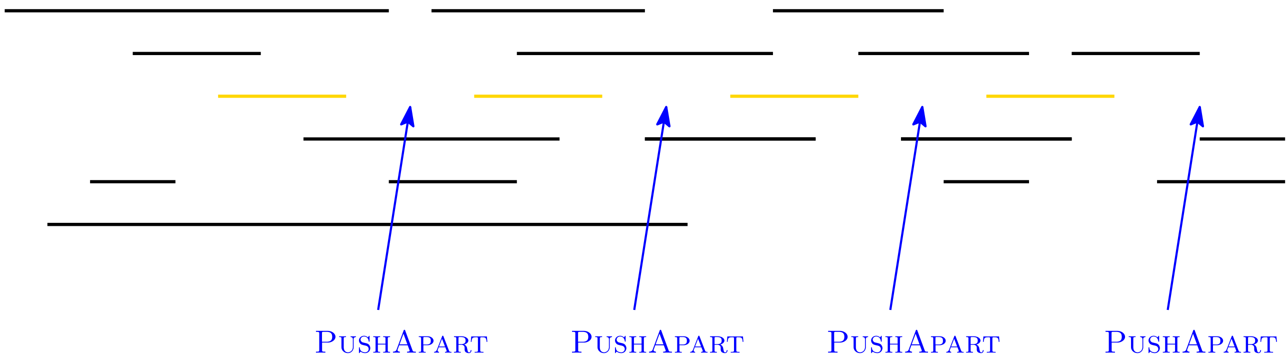


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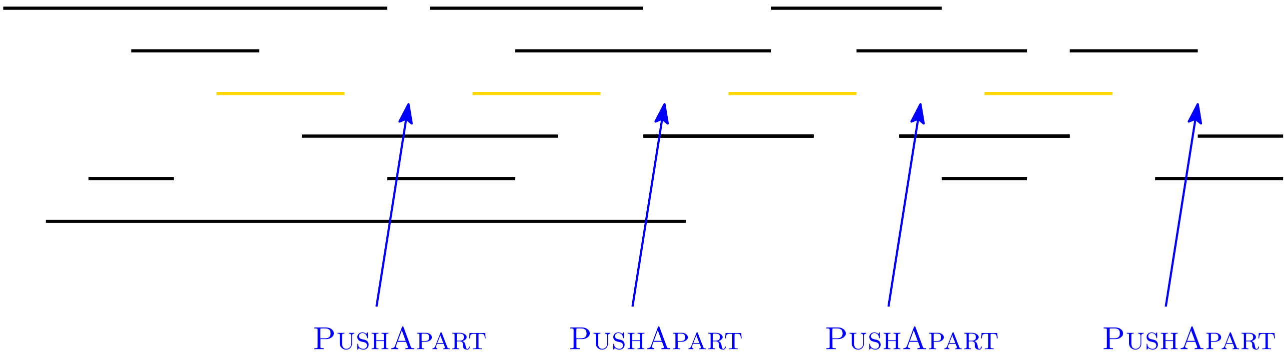


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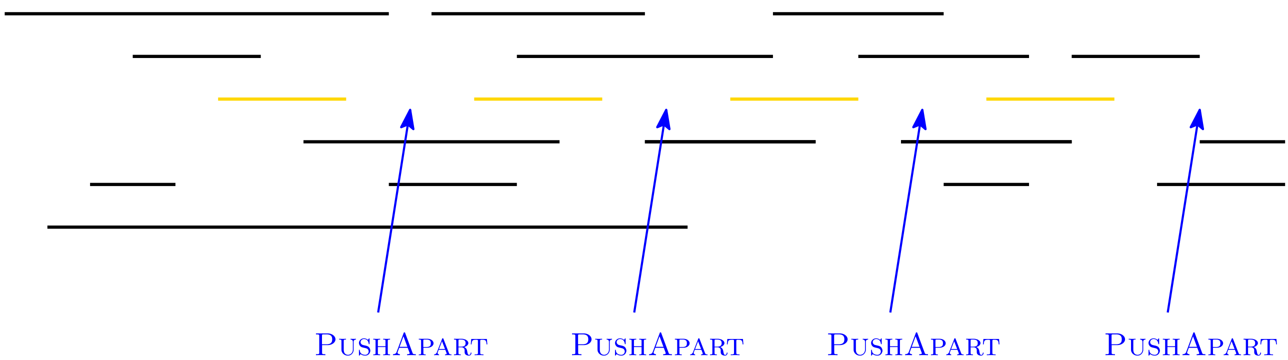


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Better order of PushApart and more careful analysis

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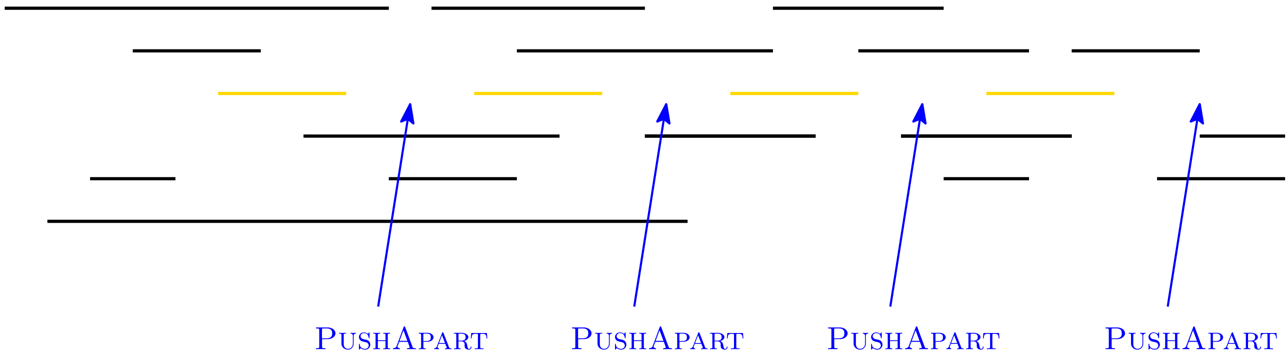


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- Bound $\mathcal{O}(kn^2)$ for the path length!

Lower bound?

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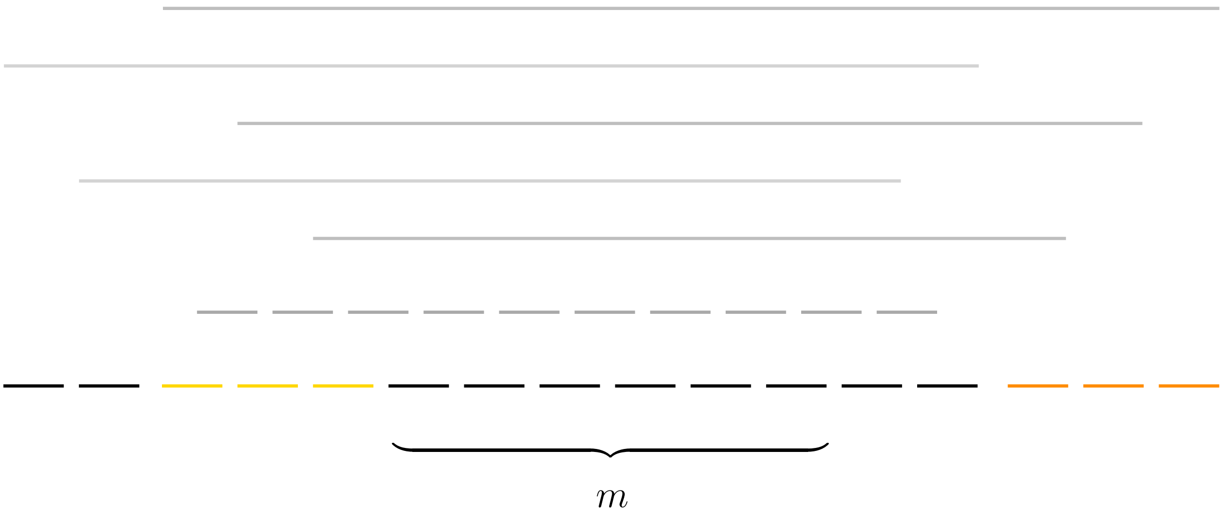

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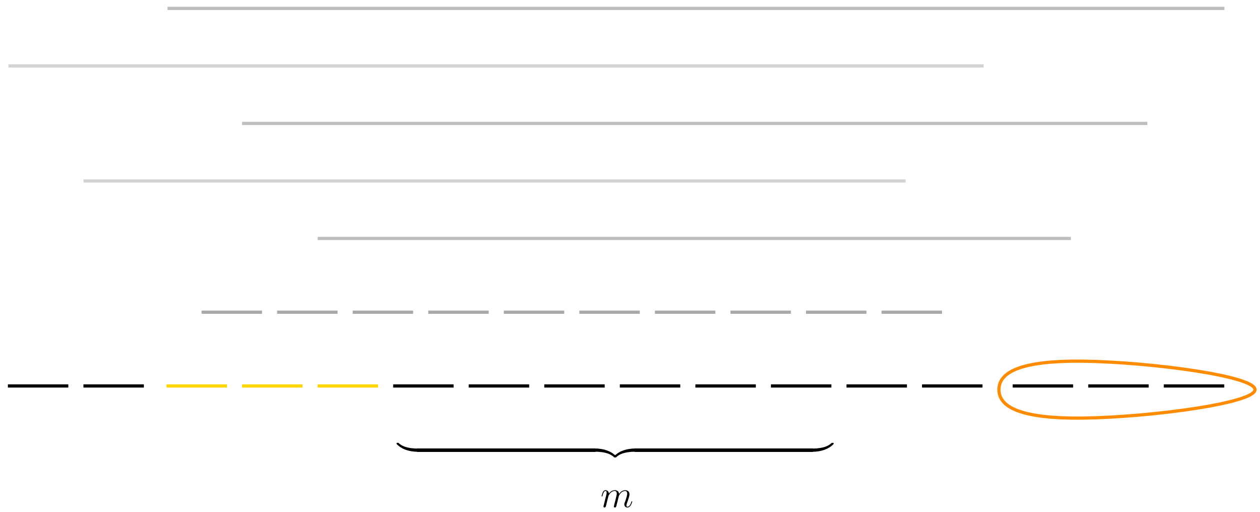

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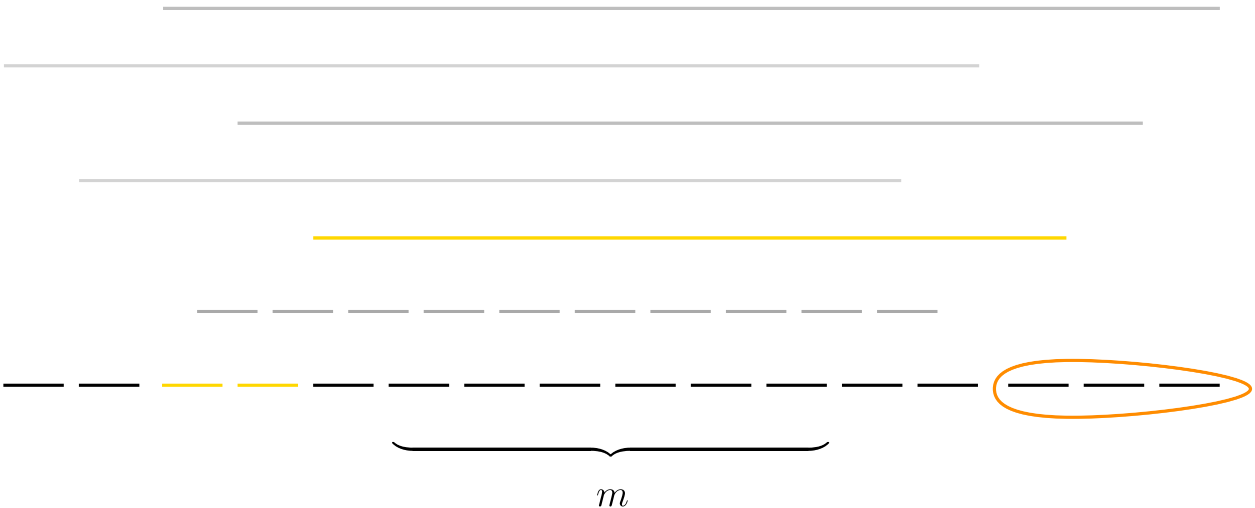

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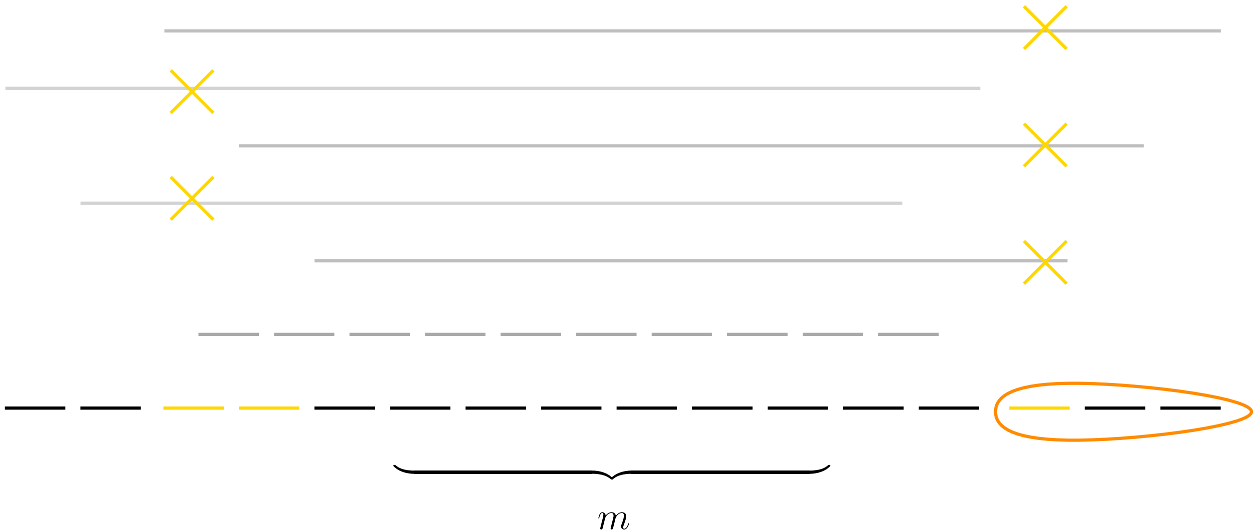

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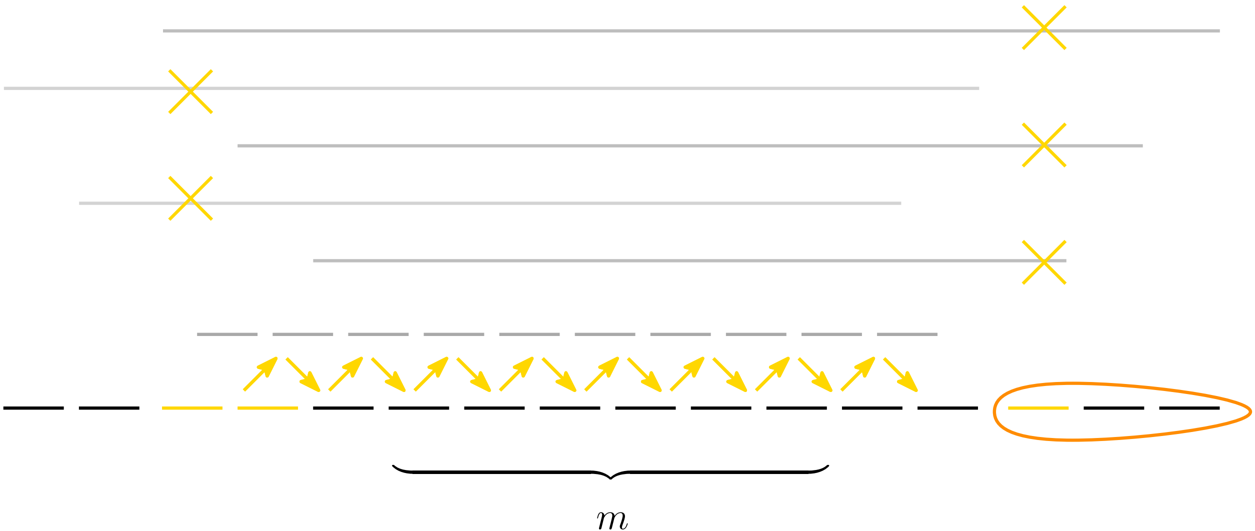

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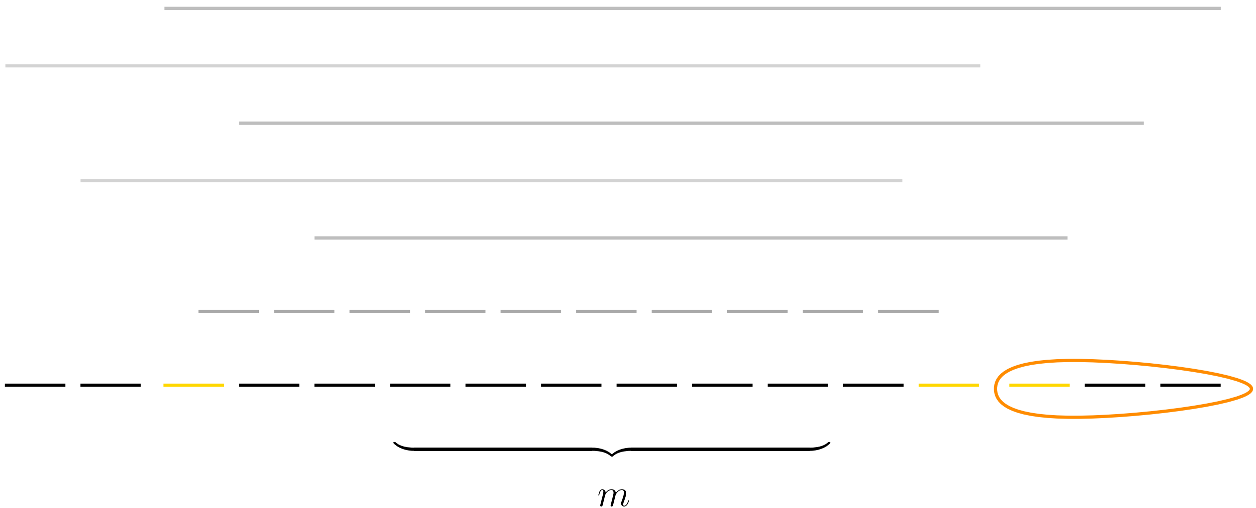

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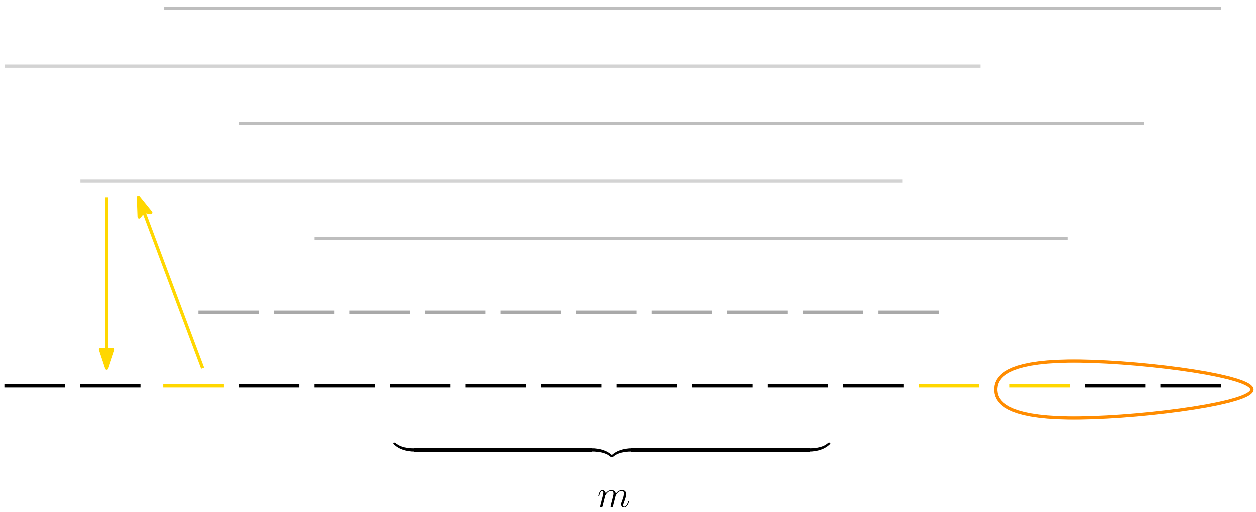

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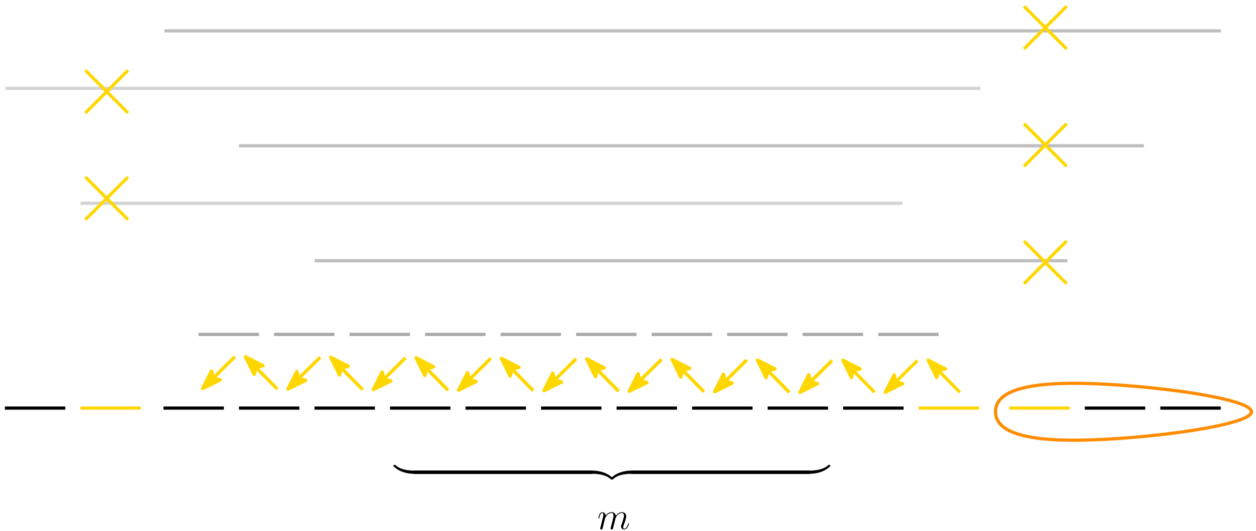

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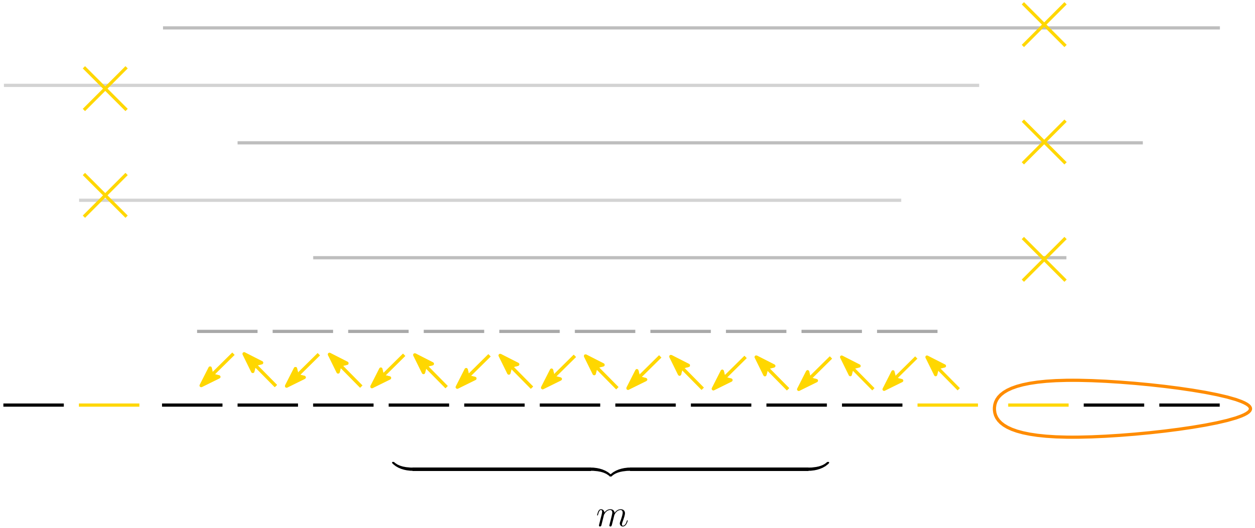

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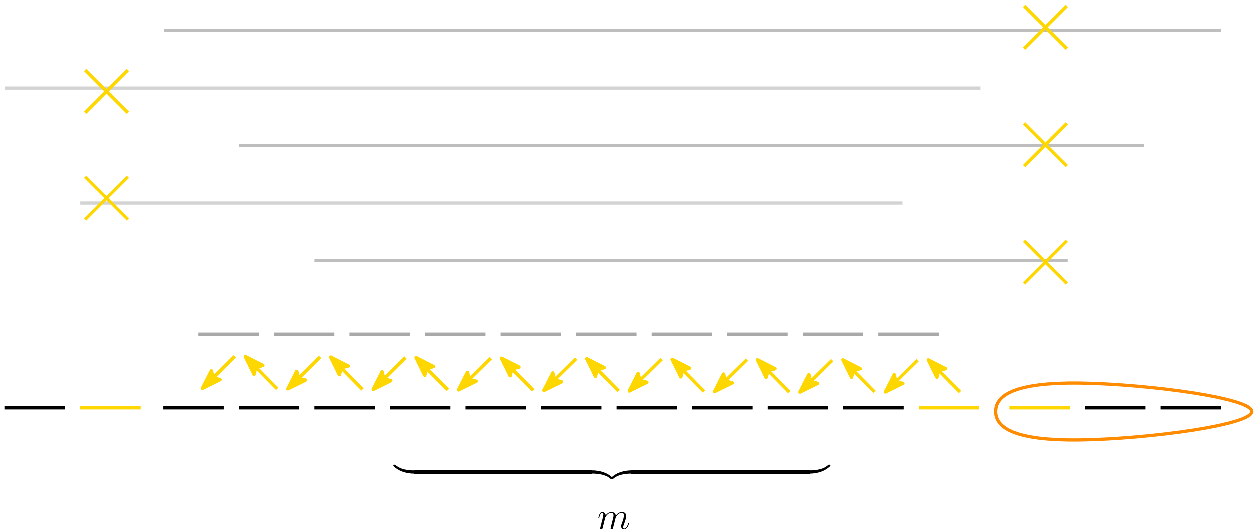

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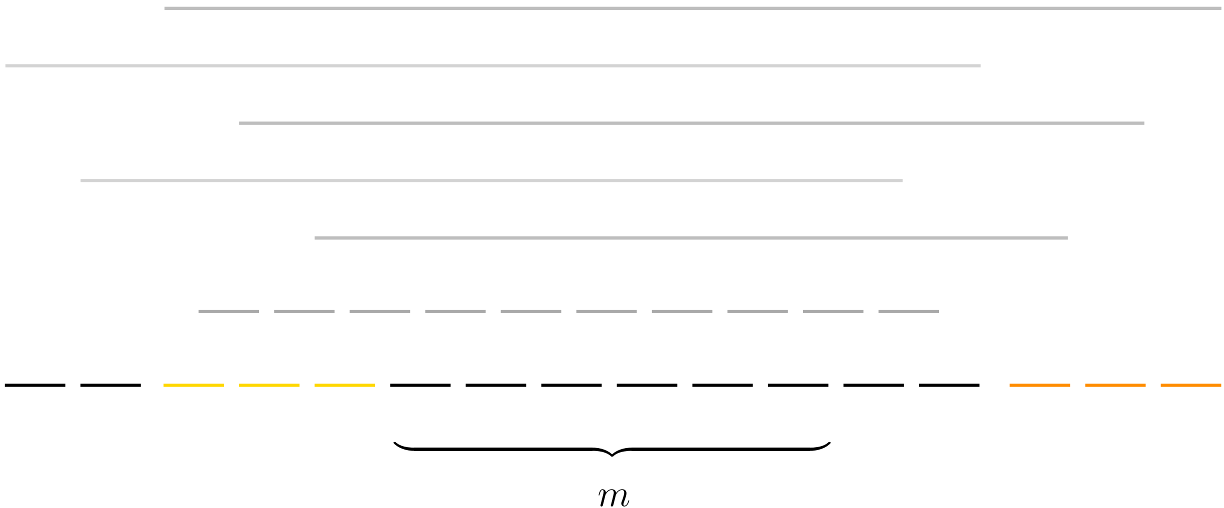

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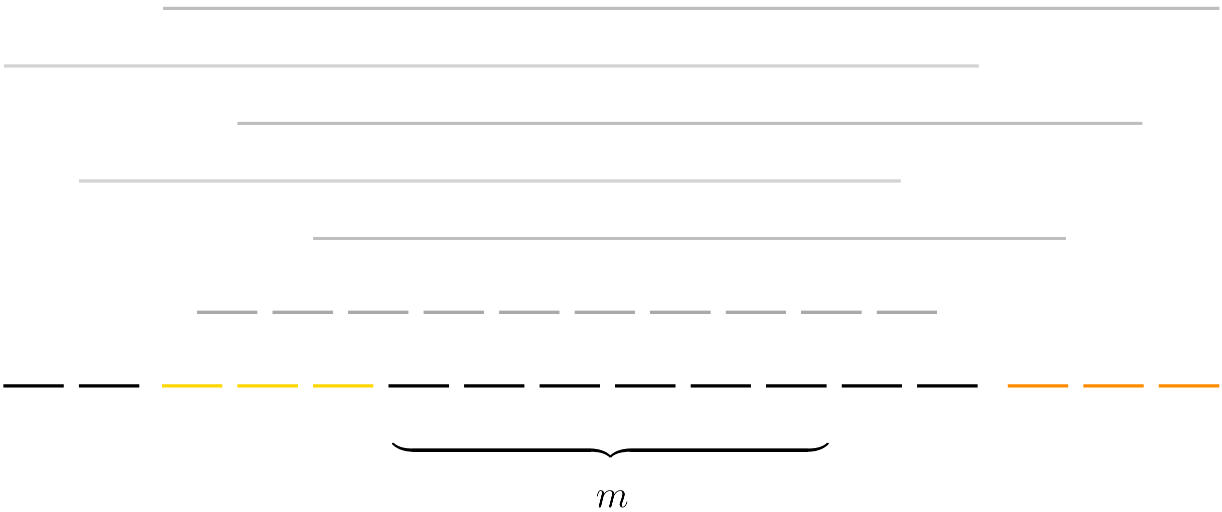
tight when $k \sim n$



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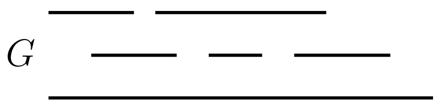

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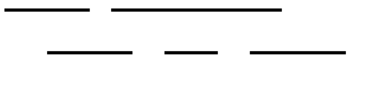
$G$



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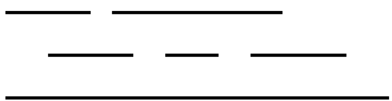

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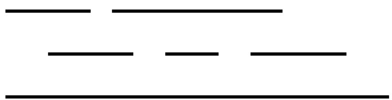

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Note: chains are independent sets in $\mathrm{Inc}(P)$

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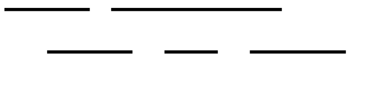

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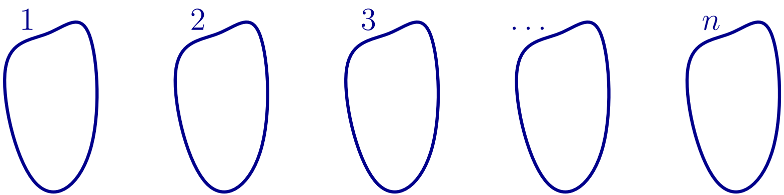


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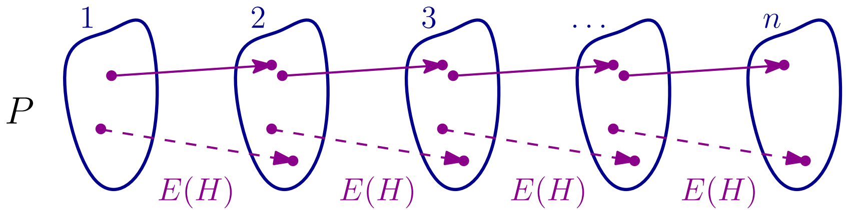


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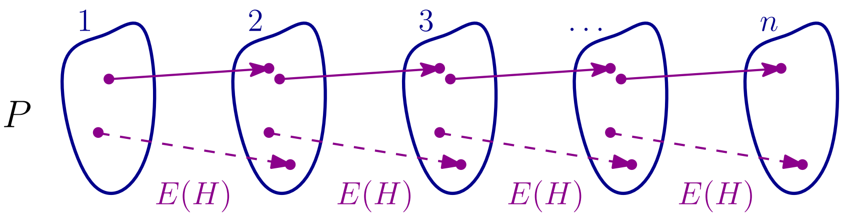


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$A := \{(a_1, 1), (a_2, 2), \dots, (a_n, n)\}$

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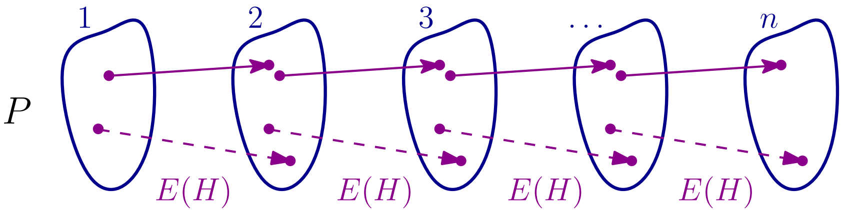
[Wrochna '18] There exists $H$ such that $H$-Word Reachability is PSPACE-complete




$n = |a| = |b|$ copies of $V(H)$

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independent sets in $\text{Inc}(P)$

## $H$-Word Reachability reduction

$H$ - digraph $\qquad a = a_1 a_2 \ldots a_j \in V(H)^*$

$a$ is an <u>$H$-word</u> if $a_i a_{i+1} \in E(H)$ for all $i$

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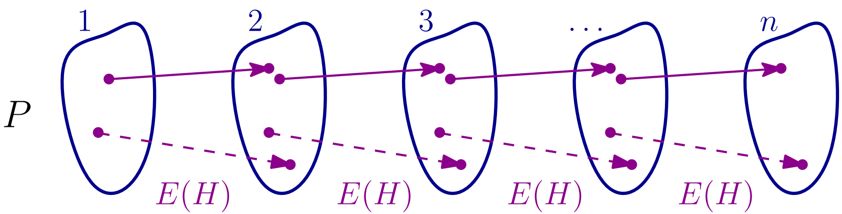


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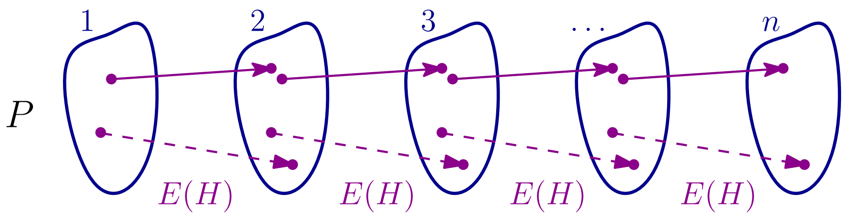


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intermediate words are $H$-words

$\Longleftrightarrow$

intermediate sets are independent sets